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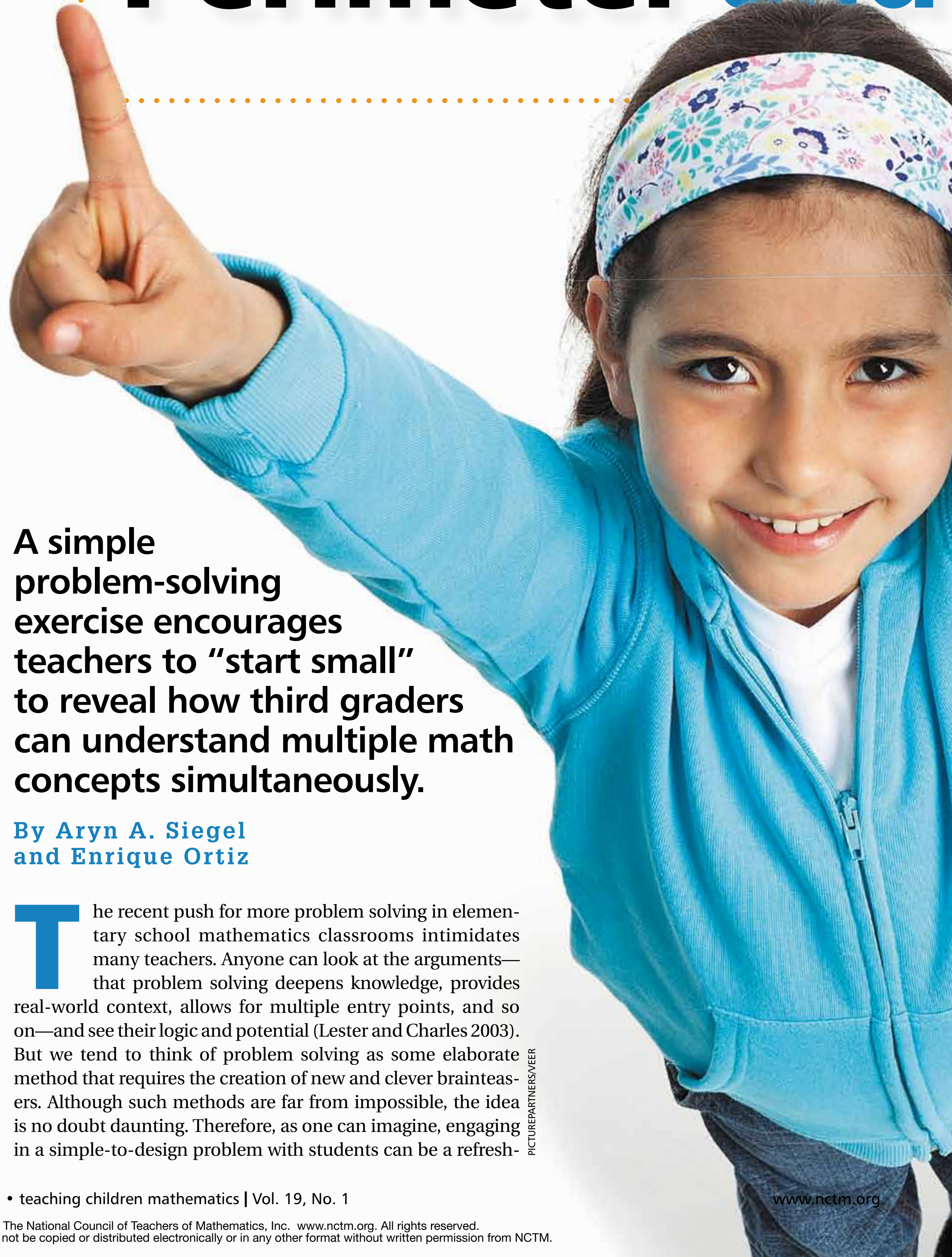

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A simple problem-solving exercise encourages teachers to "start small" to reveal how third graders can understand multiple math concepts simultaneously.

By Aryn A. Siegel and Enrique Ortiz

The recent push for more problem solving in elementary school mathematics classrooms intimidates many teachers. Anyone can look at the arguments—that problem solving deepens knowledge, provides real-world context, allows for multiple entry points, and so on—and see their logic and potential (Lester and Charles 2003). But we tend to think of problem solving as some elaborate method that requires the creation of new and clever brainteasers. Although such methods are far from impossible, the idea is no doubt daunting. Therefore, as one can imagine, engaging in a simple-to-design problem with students can be a refresh-

Beyond

ing, emboldening experience as well as proof that problem solving does not necessarily *have* to be complex and planning-intensive to be incredibly powerful and illustrative.

I (Siegel) started a lesson thinking I was going to be reviewing concepts of perimeter with my third-grade students. For them, the concept was straightforward: Add the lengths of a polygon's sides, and you have it. But what started as a routine review ended up revealing a lot more about my students than I originally expected.

We began with a simple rectangle. The length (8 cm) and width (2 cm) were labeled, but on two sides only (see **fig. 1**), requiring students to use the two sides that were labeled to determine the perimeter of the entire rectangle. Because some students had struggled with naming quadrilaterals, I first asked the class to name the polygon. I saw this as an opportunity to revisit this vital math concept (Goldenberg, Shteingold, and Feurzeig 2003). Students easily recalled the terms *rectangle* and *quadrilateral*. The real discussion came after I asked if the polygon had any other possible names. One student immediately chimed in with *rhombus*. When I asked the others to vote with thumbs up for agreement and thumbs downs for disagreement, the class was somewhat evenly split. I gave the students a few minutes to discuss their reasoning with a partner, and then we came back together to discuss it as a group. By this time, we were in agreement that we could not call this polygon a rhombus or a square, because it did not have four sides of equal length. However, the class agreed that we *could* call the polygon a *parallelogram*, as it had two pairs of parallel sides. Once that issue was settled, I asked my students to solve the problem.

What is the polygon's perimeter?

Having students work independently at first, I was not surprised that each and every one of them immediately drew a picture to aid them. Pictorial representation is an incredibly important tool (Kelly 1999) that my students are encouraged to implement on a near-daily basis. By using their diagrams, almost everyone

FIGURE 1

The third graders had previous experience only with polygons for which all sides were labeled.

"And how did you get *your* answer?"

was able to determine the missing side lengths. The discussion of permissible names for the polygon seemed influential in this endeavor. As mentioned, the students had spent the minutes immediately prior to the problem reviewing the requirements of a rectangle. So they did not labor with this aspect of the problem. However, they were required to apply our discussion to their solutions. From there, I assumed that students would simply add 2 + 2 + 8 + 8 to determine the perimeter, 20 cm.

When I asked students to share their solutions with a partner, I was again astonished at how this simple problem was opening a window into my students' understanding of different mathematical ideas. Some, of course, used the simple addition of the side lengths, but even then they used different strategies. Students added the lengths of the two "short sides" and the two "long sides" and then combined the sums using traditional algorithms (see **figs. 2a** and **2b**). Other students completed a similar process, but they needed a physical representation of the concepts, so they physically paired the like sides by circling them and then combining them using addition (see **figs. 3a** and **3b**).

A third novel idea for addition surfaced. A student said that she "looked for tens." When asked to explain, she told us that she thought adding numbers when they are in tens is "really easy" and she had noticed that "two and eight would make one ten, and the other two and eight made another ten." She then combined the two tens to find the perimeter of 20 cm (see **fig. 4**), dazzling her classmates, not to mention her teacher. I had not thought about teaching this strategy for addition, although it is one that adults often use without thinking about it. Now my students have been using this for any number of addition computations, not only for perimeter situations.

The final strategy the third graders presented was actually the one that the lesson was intended to introduce. For this strategy, the student multiplies the length by two and the width by two and then adds the resulting products (see **fig. 5**). Asked to explain his reasoning, one student shared, "I know that there are two of the short side and two of the long side, and they're the same." He reasoned, "Since there are two that are the same, I can just double them and then add them up for the perimeter." I could easily have stood by the whiteboard, lectured my students, and explained this method to them. Truthfully, I must admit it was my original plan. Imagine my surprise when I heard the strategy from a student. I did not need to belabor the topic; it made so much more sense to students when it came from a peer. A majority of them have internalized this method and now use it regularly.

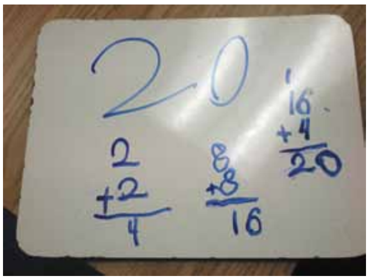
Conclusion

Although this one problem and its discussion and application of problem solving consumed over half that day's math period, it was well worth our time. What started as a seemingly simple perimeter question turned into a session of real mathematical thinking and discussion, facilitated mostly by my asking the simple question, "And how did you get *your* answer?" Students were able to debate and rationalize

FIGURE 2

A simple problem opened windows into students' different strategies for the same mathematical ideas.

(a) One method is to add the two short sides of a polygon, then the two long sides, and then combine the sums.



(b) Another is to add all four sides to find its perimeter.

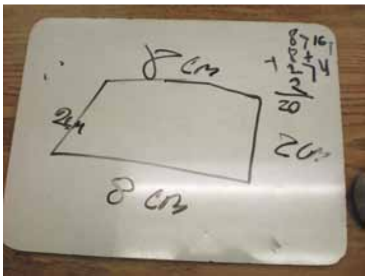


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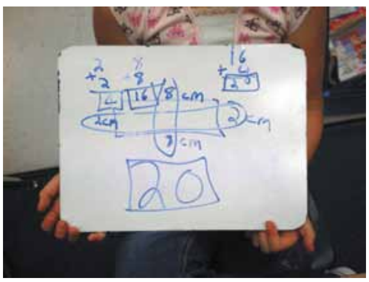
FIGURE 3

Some students needed a physical representation, so they circled the sides that were alike before adding them.

(a)



(b)

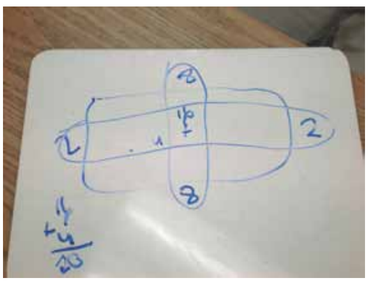


FIGURE 4

A student impressed her teacher and classmates with a making-tens strategy; tens are easy numbers to add.

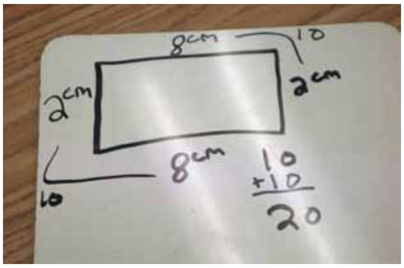
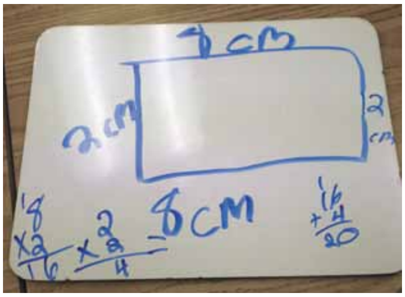


FIGURE 5

When a peer finds a strategy, students retain more knowledge than when they are lectured to or taught algorithms.



geometry, number sense, addition, multiplication, and reasoning concepts. They learned from one another, generalized, and experimented with others' methods as we moved on to increasingly complex perimeter problems.

I have repeatedly read about the power of problem solving and collaborative work. However, it took watching my students engage in such activities for me to see the true potential. I learned so much about my students' mathematical ideas, and we really explored those ideas. As an added bonus, I could see that students *enjoyed* tackling the problem and sharing their ideas. As promised, they relished the idea of discussing their problem solutions (Buschman 2003). They literally begged me not to stop the lesson. Therein lies the true power of problem solving: open questions that incite active engagement for everyone involved.

For anyone who is intimidated by the idea of problem solving, I definitely join those who advise teachers to start small (Van de Walle 2003). Leave out a key bit of information, and require students to use mathematical reasoning to discover it on their own. Ask children to share their methods with a partner, and then invite them to share with everyone. Ask, "Did anyone try a different way?" Make the commitment to try it once. You might be surprised, but you will *not* be disappointed.

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